

# Boolean Hypercubes: A Universal Application Mathematical Construct

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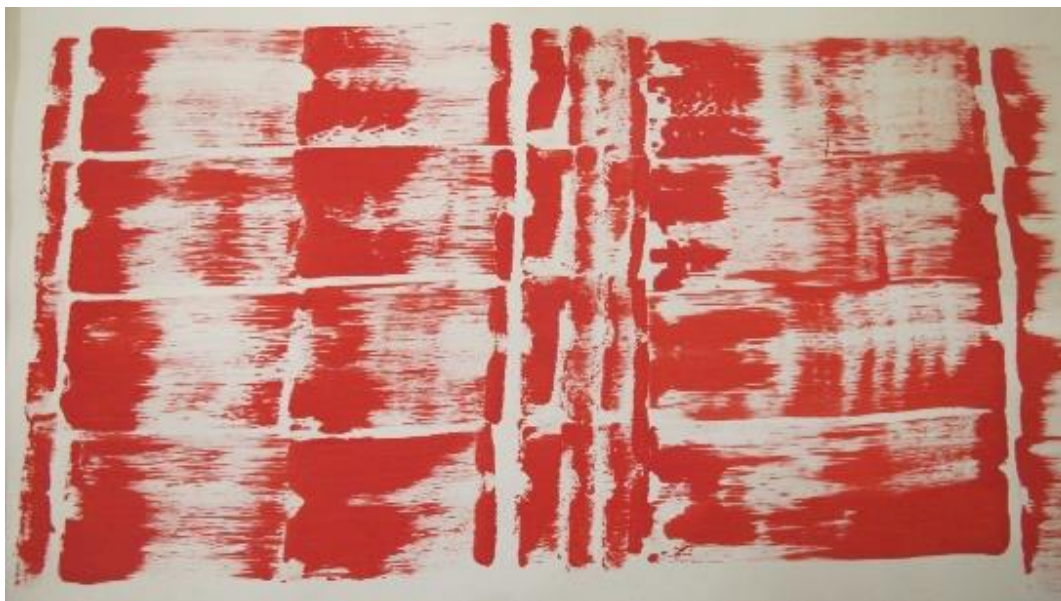


Figure by Pep Camps, Girona Painter

## Summary

Since 1997 [R. Carbó-Dorca; “Fuzzy Sets and Boolean Tagged Sets” J. Math. Chem. **22** (1997) 143-147.] Boolean Hypercubes have been one of the leitmotifs of my research at the IQCC. After this first paper, more than 30 papers have been devoted to the application of Boolean Hypercubes in chemical, physical, biological, and mathematical problems.

Apart from the fact that any human knowledge can be resumed into the vertices of a Boolean Hypercube of the appropriate dimension, one of the most characteristic among their applications is the definition of Boolean Tagged Sets, a general framework that encompasses the well-known Fuzzy Sets, which become a particular case of Tagged Sets, a generalization of the original concept.

In general, they can be applied in Quantum Similarity ordering and description of molecular sets. Tagged Sets are tools to obtain ordered structures among the elements of a given set, in particular on sets made of molecular structures with some properties attached. Once ordered, the molecules in the set can be displayed on the vertices of a conveniently projected and drawn Boolean Hypercube.

However, other applications of Boolean Hypercubes can be associated with physical problems, like the time representation and the evolution of a Fuzzy Boolean Hypercube. Synchronicity and asynchronous evolution can be studied in this way. Connection with quantum computation can be easily done.

Generalization of the concept of sign, via defining collective signatures of vectors and matrices, and the collective assignation of electron spin in MO theory can be obtained via adequate manipulation of Boolean Hypercubes.

Moreover, Boolean Hypercubes might be also associated with biochemical topics, like the modern reconstruction of DNA using an extended kind of basis, larger than the four usual biological ones, and thus promoting the information variation of the associated structure of codons.

Boolean Hypercubes can be used in the study of the chirality of peptides, as chirality is a binary property.

In some mathematical studies, Boolean Hypercubes have been used among other subjects as a background to generalize the Fermat theorem, to discuss an Erdős conjecture, and to set up the Collatz conjecture.

Mathematical Logic can be also mentioned as a subject where Boolean Hypercubes play a role, as well as in the discussion of Gödel theorems and Cantor transfinite numbers.

Vector space inner structure can be studied using Boolean Hypercubes that act as a template to construct finite-dimensional vectors. The concept of a perfect vector arises from this construction.